Multivariable Calculus
Quiz 2 Name: ____________________

1) Find the intersection of the line $\vec{r}(t) = \langle -t+4,\ 3t+1,\ 2t-3\rangle$ with the plane $2x - 3y + 4z = 5$.

Solution: Substituting the expression in the line for x, y, and z into the equation of the plane gives

$$\begin{aligned} 2(-t+4) - 3(3t+1) + 4(2t-3) &= 5 \\ -3t - 7 &= 5 \\ t &= -4 \end{aligned}$$

So, the point of intersection is on the line with parameter $t = -4$. That gives $\vec{r}(-4) = \langle 8,\ -11,\ -11\rangle$ which we interpret as the point $(8, -11, -11)$.

2) Find the line of intersection of the planes $2x - 3y + 4z = 4$ and $3x + y - 5z = 6$.

Solution: We first find any point on the line of intersection. Choosing $z = 0$ gives the equations

$$\begin{aligned} 2x - 3y &= 4 \\ 3x + y &= 6 \end{aligned}$$

which has solution $x = 2$ and $y = 0$. So, the point $P = (2, 0, 0)$ is on the line of intersection.

To get a direction vector for the line, we take the cross product of the two normal vectors.

$$\begin{aligned} \vec{n}_1 \times \vec{n}_2 &= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -3 & 4 \\ 3 & 1 & -5 \end{vmatrix} \\ &= \langle 11, 22, 11\rangle \end{aligned}$$

Since any multiple of the direction vector will work, we can take $\vec{d} = \langle 1, 2, 1\rangle$. That gives the line of intersection as

$$\vec{r}(t) = \vec{OP} + t\vec{d} = \langle t+2, 2t, t\rangle.$$

3) Do the lines $\vec{r}_1(t) = \langle -t+4,\ 3t+1,\ 2t-3\rangle$ and $\vec{r}_2(s) = \langle 2s-1,\ s+2,\ -s+1\rangle$ intersect? If so, at what point?

Solution: We can take the expressions for x in y in the two lines and set them equal. This typically produces a candidate pair of values for t and s. We can then substitute these values into their respective lines to see if they produce the same point (this is equivalent to making sure the values for z match).

The system

$$\begin{aligned} -t + 4 &= 2s - 1 \\ 3t + 1 &= s + 2 \end{aligned}$$

has solution $t = 1$ and $s = 2$. Substituting these values into the lines gives

$$\begin{aligned} \vec{r}_1(1) &= \langle 3,\ 4,\ -1 \rangle, \\ \vec{r}_2(2) &= \langle 3,\ 4,\ -1 \rangle. \end{aligned}$$

So, the lines intersect at the point $(3, 4, -1)$.